Input Noise Current Spectral Density Estimation for a Distributed Based Optical Receiver

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Abstract

Modelling of the input noise current spectral density of the distributed amplifier (DA) based optical receiver is carried out and an expression of the input noise current spectral density is presented, showing the effect of the photodiode impedance. Derived analytical expressions show the contribution of the different noisy elements to the overall noise.

1 Introduction

THE use of distributed amplifiers with flat gain response within optical receivers is very attractive since the topology of such amplifiers allows improved utilisation of the bandwidth provided by the active devices used [1]. Also, DAs have recently been constructed as transversal filters (TF) [2] to effectively shape/filter the travelling wave signals where amplification and equalisation are combined in a single circuit.

Distributed amplifiers suffer from higher noise figures compared with those produced by other amplifier topologies, such as high impedance and transimpedance topologies. Accordingly, analysis of the DA noise characteristics is required. Although noise in DAs has been extensively studied [3, 4], a detailed study of the noise for optical receivers based distributed amplifiers is not available. In [4] the photodiode is embedded in a T-junction thus forming the first stage of the gate artificial line and matching the formed T-junction impedance to the gate artificial transmission line impedance. Recently, Freundorfer et al. [5] gave an approximate value for the input noise spectral density in distributed based optical receivers by assuming that the image impedances of the artificial transmission lines and their characteristic impedances are the same. Here an accurate derivation of the noise spectral density, $< i^2 >$, at the input of the front-end distributed amplifier, considering the impedance of the photodiode, is outlined. In this analysis a simple DA model based on that proposed by Aitchison [3] is used. Calculated results of the noise spectral density using the analytical expression show the influence of the different amplifier parameters on $< i^2 >$.

2 The Optical Receiver Distributed Amplifier

Modelling the noise current at the input of an optical receiver preamplifier is usually achieved by calculating the contribution of all individual noise generators in the different amplifier stages, and then referring the corresponding noise spectral densities to the input. For a distributed amplifier, with a non-resistive input termination, as in the case for a photodetector – DA configuration, this process is more complicated than it is for conventional amplifier structures, as there are propagating and counter propagating waves on the gate artificial transmission line. This necessitates a wave-based analysis of the noise of the DA.

The simplified equivalent circuit of a n-stage distributed amplifier based optical receiver ignoring artificial line losses is shown in Figure 1 (without the networks in the dashed boxes). At the drain terminations and at the right-hand side gate termination, it can easily be seen that there is an impedance mismatch, Z_d and Z_{dT}, and Z_g and Z_{gT}, respectively. Z_g and Z_d are the lossless gate and drain impedance terminations given by $\sqrt{L_g/C_{gs}}$ and $\sqrt{L_d/C_{ds}}$, respectively, and Z_{gT} and Z_{dT} are the constant k gate and drain T image impedances given by $Z_{gT} = Z_g\sqrt{1-(f/f_c)^2}$ and $Z_{dT} = Z_d\sqrt{1-(f/f_c)^2}$ [6], respectively, where f is the frequency and f_c is the equalized cutoff frequency of the gate and drain line given by $f_c = 1/(\pi\sqrt{L_gC_{gs}}) = 1/(\pi\sqrt{L_dC_{ds}})$. C_{ds} is the cumulative drain to source capacitance which consists of the active device drain-to-source capacitance in parallel with the additional capacitance used to equalize the phase velocities of the gate and drain lines. This mismatch is conventionally remedied by the insertion of m-derived L sections (see dashed boxes in Figure 1). These L sections have T image impedances equal to the T image impedance of the artificial lines in which they are embedded in and have π image impedances given by [6]

$$Z_{d,g_{m\pi}} = Z_{d,g}\frac{1-(f/f_o)^2}{\sqrt{1-(f/fc)^2}}, f_o^2 = \frac{f_c^2}{1-m^2} \quad (1)$$

For m = 0.6 $Z_{d,g_{m\pi}} = Z_{d,g}$ for $f < f_c$. Therefore by inserting these m-derived (m=0.6) L sections we get matching at all terminations except at the photodiode port. It is the effect of the impedance mismatch at the photodiode port on the noise performance that will be treated in this paper.

As will be shown later, all noise generators current spectral densities are calculated at the output of the DA and then referred to the input by straightforward division by the square of the magnitude of the current gain. In this section the current in the drain terminating impedance at the output of the DA is derived. Also, the transimpedance and current gains of the DA are derived.

In Figure 1, the current going through the right hand side drain impedance Z_d is given by:

$$I_{dr} = \frac{1}{2}g_m e^{-j\beta_d/2}e^{-j\beta_{m_d}/2}\sum_{k=1}^{n} V_k e^{-j(n-k)\beta_d} \quad (2)$$

and the voltages across the gate-to-source capacitances are given by:

$$V_k = V_{in}\frac{Z_{g_\pi}}{Z_{gT}}e^{-j(k-\frac{1}{2})\beta_g} \quad (3)$$

where β_d and β_g are the drain and gate artificial transmission line phase constants given by $\omega\sqrt{L_d C_{ds}}$ and $\omega\sqrt{L_g C_{gs}}$, respectively, and $\beta_{m_d}/2$ and $\beta_{m_g}/2$ are the phase constants of the m-derived drain and gate L sections, respectively. Substituting eq. (3) into eq. (2) we get, for the flat transimpedance gain case (phase synchronised lines $\beta_d = \beta_g = \beta$)

$$I_{dr} = \frac{1}{2}ng_m\frac{Z_{g_\pi}}{Z_{gT}}V_{in}e^{-jn\beta}e^{-j\beta_{m_d}/2} \quad \text{and}$$

$$Z_{dg} = \frac{1}{2}ng_m Z_{g_\pi}Z_d e^{-jn\beta}e^{-j\beta_{m_d}/2} \quad (4)$$

For the T networks the mid-shunt gate and drain π impedances are given by

$$Z_{g_\pi} = \frac{Z_g}{\sqrt{1-(f/f_c)^2}}, Z_{d\pi} = \frac{Z_d}{\sqrt{1-(f/f_c)^2}} \quad (5)$$

The output to input current ratio is given by:

$$A_I = \frac{I_{dr}}{I_{in}} = \frac{V_{dr}/Z_d}{I_{in}} = \frac{Z_{dg}}{Z_d} \quad (6)$$

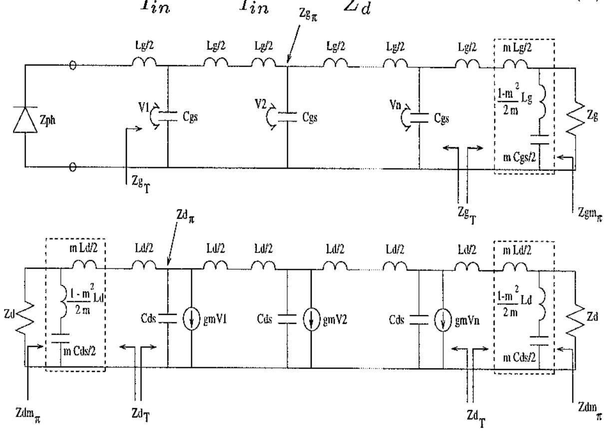

Figure 1: Distributed Based Optical Receiver Simplified Equivalent Circuit

3 Noise Modelling

For an amplifier like the one described above, the noise current generators are as follows: (i) The gate load impedance Z_g at temperature T_o, (ii) the left hand side drain load impedance Z_d at T_o and (iii) noise associated with each of the n FETs (i_g and i_d noise sources parallel with C_{gs} and C_{ds} respectively) [4]. The thermal contribution of the right hand side drain load impedance Z_d will be ignored in our modelling as it is part of the following network. In practice, the photodiode model will incorporate a series resistor which generates noise. Also this noise is not dealt with here but could be added easily. Figure 2 shows all the noise generators associated withe the DA noisy elements. In what follows, each of the noise sources will be treated separately.

Noise caused by the gate line terminating impedance

The noise current spectral density of the gate line impedance is $< i_{Z_g}^2 >= 4kT/Z_g$. We calculate the rms value due this current at the output of the DA. In order to do so we calculate first the current in the output of the drain line caused by the propagation of the noise current, given by, $\frac{Z_g}{Z_g+Z_g}i_{Z_g} = \frac{1}{2}i_{Z_g}$ in the gate line towards the input. We shall represent this current by I_{d1}. Since the source features the impedance of the photodiode Z_{ph} which is different from Z_g, the noise current is reflected back towards the gate line termination impedance and subsequently absorbed. This reflected current will again induce a current in the drain line and part of this current will propagate to the output drain impedance. We represent this current by I_{d2}.

$$I_{d1} = \frac{1}{4} i_{Z\,g}\, g_m\, Z_{g\,\pi}\, e^{-j(\beta_{m\,g}+\beta_{m\,d})/2} \\ e^{-j(n+\frac{1}{2})(\beta_g+\beta_d)} \sum_{r=1}^{n} e^{j(\beta_g+\beta_d)r} \quad (7)$$

for $\beta_d = \beta_g = \beta$ we get:

$$I_{d1} = \frac{1}{4} i_{Z\,g} g_m Z_{g\,\pi} \frac{\sin(n\beta)}{\sin(\beta)} e^{-j(n\beta+(\beta_{m\,g}+\beta_{m\,d})/2)}$$

Now for the reflected noise component

$$I_{d2} = \frac{1}{4}\Gamma i_{Z\,g} g_m Z_{g\,\pi} \Big(e^{-j(n-1+\frac{1}{2})\beta_d} e^{-jn\beta_g} e^{-j\frac{1}{2}\beta_g} + \cdots \\ + e^{-j(\frac{1}{2}\beta_d+n\beta_g)} e^{-j(n-1+\frac{1}{2})\beta_g} \Big) e^{-j(\beta_{m\,g}+\beta_{m\,d})/2}$$

where Γ is the source reflection coefficient defined as $\Gamma = \frac{Z_{ph}-Z_{gT}}{Z_{ph}+Z_{gT}}$. Hence If $\beta_d = \beta_g = \beta$ we get:

$$I_{d2} = \frac{1}{4} i_{Z\,g} \Gamma g_m Z_{g\,\pi} n e^{-j2n\beta} e^{-j(\beta_{m\,g}+\beta_{m\,d})/2} \quad (8)$$

The noise spectral density caused by the gate line termination impedance, at the output of the DA can now be calculated:

$$< i_{Z\,g}^{\,2} >_o = < (I_{d1}+I_{d2})(I_{d1}+I_{d2})^* > \quad (9)$$

This can be referred to the input as

$$< i_{Z\,g}^{\,2} >_i = \frac{< i_{Z\,g}^{\,2} >_o}{|A_I|^2} = \frac{Z_d^2 < i_{Z\,g}^{\,2} >_o}{|Z_{dg}|^2} \\ = \frac{1}{4} < i_{Z\,g}^{\,2} > \Big[\frac{\sin^2(n\beta)}{n^2 \sin^2(\beta)} \\ + 2\frac{\sin(n\beta)}{n\sin(\beta)} \mathrm{Real}\left(e^{jn\beta}\Gamma^*\right) + |\Gamma|^2 \Big] \quad (10)$$

Noise caused by the left-hand side drain line terminating impedance

The thermal noise current generator of the left hand side drain line impedance has a spectral density of $< i_{Z\,d}^{\,2} >= 4kT/Z_d$. This noise propagates towards the load giving the rms value at the output of the DA of

$$I_d = \frac{1}{2} i_{Z\,d}\, e^{-jn\beta_d} e^{-j\beta_{m\,d}} \quad (11)$$

Therefore, the noise spectral density at the output of the DA is given by:

$$< i_{Z\,d}^{\,2} >_o = < I_d\, I_d^{\,*} >= \frac{1}{4} < i_{Z\,d}^{\,2} >= \frac{kT}{Z_d} \quad (12)$$

The noise spectral density associated with this noise source can now be referred to the input as:

$$< i_{Z\,d}^{\,2} >_i = \frac{< i_{Z\,d}^{\,2} >_o}{|A_I|^2} = \frac{< i_{Z\,d}^{\,2} >}{4} \left(\frac{Z_d}{|Z_{dg}|}\right)^2 \quad (13)$$

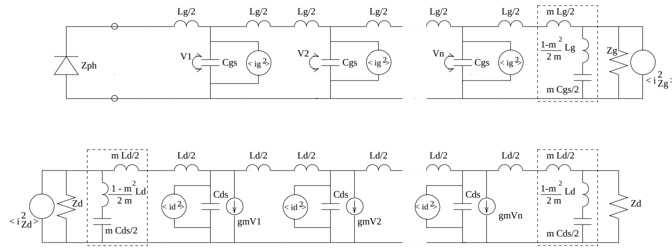

Figure 2: Noise Sources in the Distributed Based Optical Receiver

Noise Associated with the FETs

The FET noise behaviour is represented by a gate current generator and a drain current generator and a correlator current between the gate current and the drain current. These are given by [7]: $< i_g^2 >= 4kT_o \frac{(\omega C_{gs})^2}{g_m} R$, $< i_d^2 >= 4kT_o g_m P$ and $< i_g i_d^* >= j4kT_o(\omega C_{gs})P\sqrt{RC}$. P, R, and, C are constants dependent on the FET type, geometry and bias. These parameters are used to model the noise associated with the FETs as described below.

Noise Contribution due to the r^{th} FET

The current through the drain load, $I_d'(r)$, due to the r^{th} gate noise generator due to the *forward* propagation (towards the output) can be derived as before and is given for $\beta_d = \beta_g = \beta$ by

$$I_d'(r) = \frac{1}{4} g_m i_{gr} Z_{g\,\pi}(n-r+1) e^{-j((n-r+\frac{1}{2})\beta+\beta_{m\,d}/2)}$$

Here, i_{gr} represents the rms value of the r^{th} gate noise generator, that is, $\sqrt{< i_g^2 >}$.

The current through the drain load due to the r^{th} gate noise generator due to the *reverse* propagation can be decomposed into two currents. The first one considered here, $I_{d1}''(r)$, is caused by the r^{th} gate noise current, $1/2 i_{g_r}$, which propagates down the gate line towards the input. Since the source features the photodiode impedance Z_{ph} the noise current is reflected back and propagates through the gate line and is finally absorbed by the terminating gate impedance. This reflected current will induce a current in the drain line which is represented here by $I_{d2}''(r)$. For $\beta_d = \beta_g = \beta$ these currents are given by:

$$I_{d1}''(r) = \frac{1}{4} g_m i_{gr} Z_{g\,\pi} e^{-j(n+\frac{1}{2})\beta} \\ \frac{\sin[(r-1)\beta]}{\sin(\beta)} e^{-j\beta_{m\,d}/2}$$

$$I_{d2}''(r) = \frac{\Gamma}{4} g_m i_{gr} Z_{g\pi} n e^{-j(n+r-\frac{1}{2})\beta} e^{-j\beta_{md}/2}$$

For $\beta_d = \beta_g = \beta$ the current through the drain load due to the r^{th} drain noise generator is given by:

$$I_d'''(r) = \frac{1}{2} i_{dr} \, e^{-j(n-r+\frac{1}{2})\beta} e^{-j\beta_{md}/2}$$

The total current on the right drain load of the DA due to the FETs noise sources can be expressed as follows:

$$I_{do} = \sum_{r=1}^{n} [\, I_d'(r) + I_{d1}''(r) + I_{d2}''(r) + I_d'''(r) \,]$$

The noise spectral density referred to the input of the DA is given by:

$$< I_d I_d^* >_i = \frac{< I_d I_d^* >_o}{|A_I|^2} \quad (14)$$

Using eqs. (10), (13), and (14), the total noise current spectral density at the input of the DA is then given by:

$$< i^2 >_i = < i_{Z_g}^2 >_i + < i_{Z_d}^2 >_i + < I_d^2 >_i (15)$$

4 An Illustrative Example

Six MESFETs ($n = 6$) with $C_{gs} = 0.134pF$, $g_m = 14.9mS$ and the three noise parameters $P = 1.153$, $R = 0.1474$, and $C = 0.775$ were used in the implementation of an optical receiver based on a distributed amplifier suitable for operation at 10 Gbit/s [1]. The gate and drain termination impedances $Z_g = 75\Omega$, $Z_d = 50\Omega$, and a high speed photodiode ($C_{ph} = 145fF$) were also selected. Using the above derived equations, Figure 2 shows the noise contribution of the different parameters as well as the total noise. As can be seen from Figure 3, the dominant noise at low frequencies is the one caused by the gate terminating impedance whereas at high frequencies the dominant noise is that caused by the FETs.

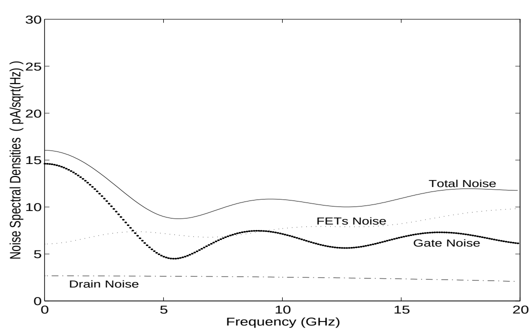

Figure 3: Input Noise Spectral Densities for the Distributed Based Optical Receiver

This makes the equations derived here useful for prediction of the noise in distributed based optical receivers at microwave frequencies and will aid designers to select appropriate devices and line impedances for a particular application.

5 Conclusion

We have presented analytical expressions for the input noise current spectral density for the distributed based optical receiver using a simplified model for the DA. Calculated results of the noise spectral density using the derived analytical expression showed the noise contribution of the different amplifier parameters on the overall noise.

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